Economics for Engineers: Notes

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January 2021

Introduction

This document contains my notes for the course ECN 801 at Ryerson University. Mostly from my lecture notes and the textbook *Contemporary Engineering Economics* by *Park, Zuo, and Pelot.*

Week I

Chapter 3: Time Value of Money

Economics is a social science concerned with how individuals, institutions and society make optimal choices under scarcity. Money has a *time value* as in:

- Money has earning power (earnings overtime through interest)
- Money has purchasing power (loss of value because of inflation)

Here is a list of important definitions and other terminology used in the study of economics:

∗ **Principal** is the initial amount of money involving debt or investments

∗ **Interest rate** is the cost or price of money and is expressed as a percentage rate per period.

∗ **Interest period** is a length of time that determines how frequently interest is calculated.

∗ **Number of interest periods** is the specified length of time of the transaction.

∗ **Plan for receipts or payments** yields a specified cash flow pattern over a specified length of time.

∗ **Future amount** is the results of interest over a number of interest periods.

The following are some important symbols used in economics:

* A_n is a discrete payment or receipts occurring at the end of some interest period.
* A is the end-of-period-payment in a uniform series that continues for n periods.
* i is the interest rate per period.
* n is the total number of cash flows (interest periods).
* P is the sum of money at $t = 0$.
* F is the future sum of money at the end of the analysis period.

Cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. The *End-of-Period Convention* states that all cash flows are assumed to occur at the end of interest periods.

Interest is calculated as follows:

$$i = \frac{interest\ accrued\ per\ time\ unit}{principle} \times 100\%$$

In order to calculate **simple interest**, which is interest that is only applied to the initial amount, you use the following equations:

$$I = Pni$$

$$F = P + I = P(1 + in)$$

In order to calculate **compound interest**, which is interest that includes the previous period's interest, you use the following equation:

$$F = P(1 + i)^n$$

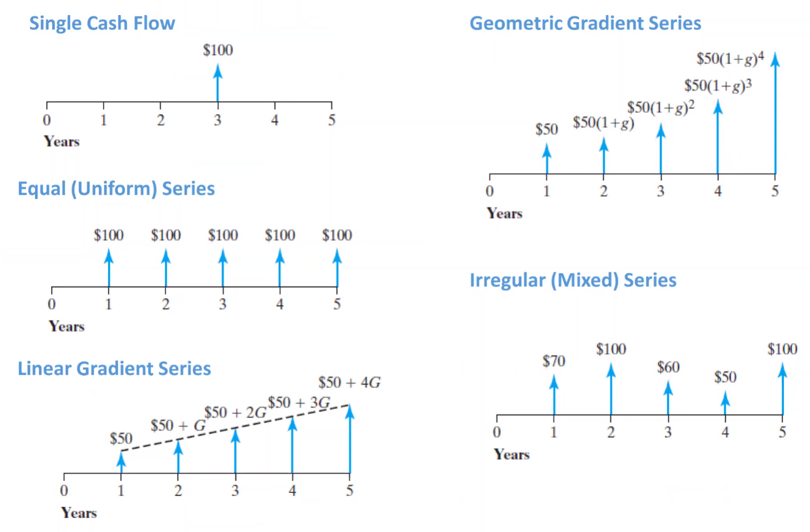
In all three equations above, **F** represents the future value, **P** represents the principal amount, **i** represents the interest rate per period, and **n** represents the number of compounding periods.

Two cash flows can be **economically equivalent** so long as you know the magnitude of the payment, the directions of the payment, the timing of the payment, and the interest rate during the period under consideration. Economic equivalence exists between cash flows that have the same economic effect, even if the timings and cash flows may differ. There are *four principles to economic equivalence*:

1. Equivalence calculations made to compare alternatives require a common time basis.
2. Equivalence depends on interest rates.
3. Equivalence calculations may require the conversion of multiple payment cash flows to a single cash flow.

4. Equivalence is maintained regardless of point of view.

Cash flow diagrams are used to show the direction of cash flow over time. An n (number of compounding periods) axis is draw horizontally with tick marks, and on each mark there can be placed an arrow pointing up or down depending on the type of cash flow. Examples are as follows:



Week II

Chapter 3: Time Value of Money ...continued

The final diagram from last week's content nicely shows us the five different **Categories of Cash Flows**, which are:

1. **Single cash flow**: Just a single present or future cash flow.

2. **Equal (uniform) series**: A series of cash flows or equal amounts at regular intervals.

3. **Linear gradient series**: A series of cash flows increasing or decreasing in a fixed amount at regular intervals.

4. **Geometric gradient series**: A series of cash flows increasing or decreasing by a fixed percentage at regular intervals.

5. **Irregular series**: No pattern overall.

I will divide each category by the case of what you know and which equations are useful.

Single Cash Flow

If you have **one cash flow**, and want to know the **future value** of that cash flow:

$$F = P\,(1+i)^n$$
$$P = F\,(1+i)^{-n}$$

Equal Payment Series

If you have **regular payments of the same amount**, and want to find the **present worth** of the money:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i, n)$$

Present Worth Factor

If you have the **present worth** of the cash flow, and want to find the **regular constant payment**:

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i, n)$$

Capital Recovery Factor

Note: here the present worth is located one period before the first payment of A.

If you have the **future value** of a fund, and want to find the **constant payment** required:

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = F(A/F, i, n)$$

Uniform Series Sharing Fund

If you have the **constant payment**, and you want the **future value of the fund**:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n)$$

Uniform Series Compound Amount Factor

Linear Gradient Series

If you have the **constant change in the cash flow**, and want the **principle amount**:

$$P_G = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right] = G(P/G, i, n)$$

Present Worth Factor: Linear Gradient

If you have the **constant change in the cash flow**, and want to know the **value of a payment**:

$$A = G\left[\frac{(1+i)^n - in - 1}{i[(1+i)^n - 1]}\right] = G(A/G, i, n)$$

Gradient-to-equal-payment Series Conversion Factor

If you have **the constant change in the cash flow**, and want the **final worth**:

$$F = G\left[\frac{(1+i)^n - in - 1}{i^2}\right] = G(F/G, i, n)$$

Declining Linear Gradient Series

Week III

Chapter 3: Time Value of Money ...continued

Geometric Gradient Series

A geometric gradient series is a series of cash flows that increase by a given percentage each interest period. g is used as the constant rate of change as a percentage.

There are three separate cases for **present worth** given **percentage increase**:

If the **interest rate is equal to the gradient percentage**, and you want the present worth:

$$P_g = A_1\left[\frac{n}{1+i}\right] \text{ only if } i = g$$

Geometric Gradient with Equal Interest and Gradient

If $i \neq g$ and **g is positive**:

$$P_g = A_1 \left[\frac{(1+i)^n - (1+g)^n}{(1+i)^n (i-g)} \right] \text{only if } i \neq g \text{ and } g > 0$$

Geometric Gradient with Positive Gradient

If $i \neq g$ and **g is negative**:

$$P_g = A_1 \left[\frac{(1+i)^n - (1-g)^n}{(1+i)^n (i+g)} \right] \text{only if } i \neq g \text{ and } g < 0$$

Geometric Gradient with Negative Gradient

Week IV

Chapter 4: Understanding Money and Its Management

There are essentially three different measures of the same interest rate. Up to this point we have been focusing on a single subsection of them without worrying about the technicalities.

The first measure is called the **annual percentage rate (APR)** also called *nominal rate*, which is the yearly cost of a loan, including interest, insurance, and the originating free, expressed as a percentage. Essentially this is what we have been considering up to this point. In the statement

18% compounded monthly

18% is the APR rate. From this value, you can divide by the number of compounding periods in the year to get the *interest rate per period*, however we will return to this later.

The next measure is **annual percentage yield (APY)** also called *effective annual interest rate*. The reason this measure is needed is because if you actually have 18% compounded monthly, you will get a slightly higher interest rate on the money. For instance:

9% compounded quarterly

The APY rate is 9.3083%. This can be calculated using the formula:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

Where:

* i_a is the effective interest rate (APY).
* r is the nominal rate.

* M is the number of compounding periods a year.

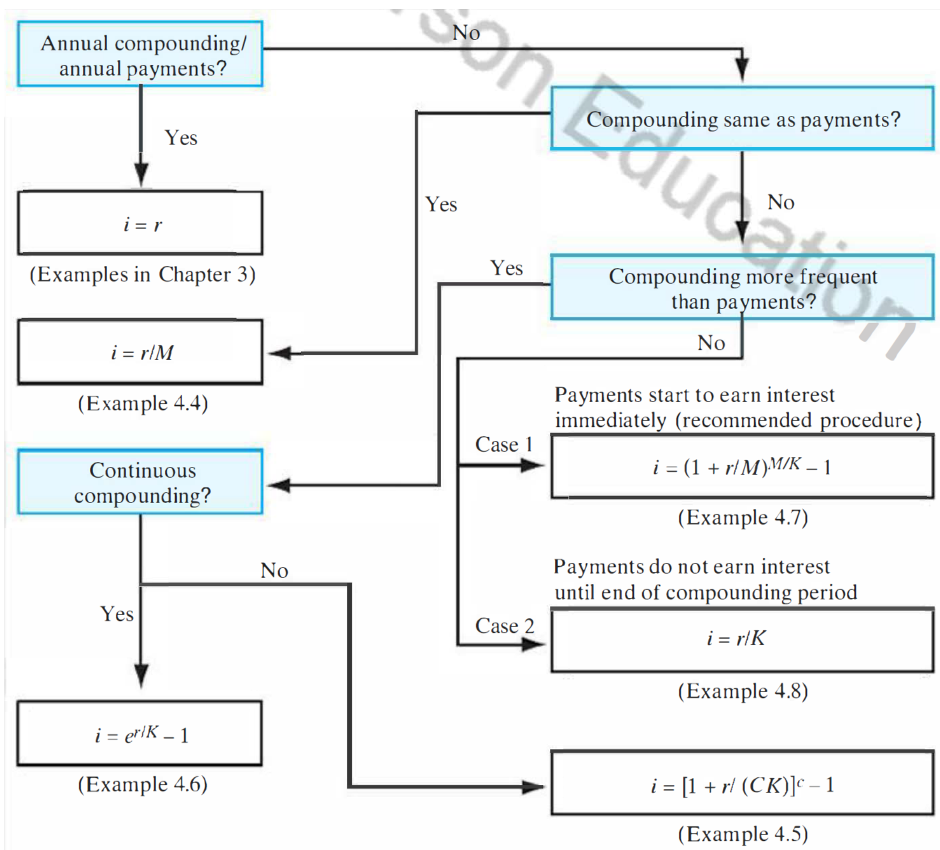
The third and final measure of interest rate for this section is **effective interest rate per payment period**. This is useful when the compounding periods and the payment periods of a payment series do not match up. Having the effective interest rate per payment period allows us to use equations like in chapter 3. (Note: in chapter 3 we only used *effective* interest rates.)

Effective interest rates per payment period can be calculated by:

$$i = \left(1 + \frac{r}{CK}\right)^C - 1$$

* i is the effective rate.
* C is the number of compounding periods per payment period.
* K is the number of payment periods per year.
* r is the nominal interest rate.

The following is a flowchart from the textbook to help you make the decision about how to approach many problems:



Week V

Chapter 4: Understanding Money and Its Management (4.5-4.6)

One of the most common types of commercial loans are **amortized loans**. This is a loan where you must repay it in periodic amounts, usually compounded monthly.

There are two main factors that effect the cost of a loan:

* The total cost of borrowing, which depends on interest rate plus fees.
* The term, or length or time it takes you to repay the loan.

The annual percentage rate (APR) is set by the lenders and include the effect of the fees. The **fees** are expenses the lender will charge to lend the money, like the application cost, the attorney fees, credit search fees, etc. **Origination fees** cover administrative costs and appraisal fees. **Periodic Interest Rate** is the interest the lender will charge on how much you borrow, without fees.

For any pay period in an amortized loan, you need to make a payment on the principal, as well as make the payment of the interest accrued on the remaining amount of the loan. This is called the *Payment Split*. Both of those can be calculated, as well as the total amount left on the loan, as follows:

$$PP_n = A[1 - (P/A, i, N - n + 1)i)] = A(P/F, i, N - n + 1) = A - I_n$$

$$I_n = A(P/A, i, N - n + 1)i = (B_{n-1})i$$

$$B_n = A(P/A, i, N - n)$$

Where:

* N is the total number of payment periods on the loan.
* PP_n is the payment on the principal for term n.
* I_n is the interest payment for term n,
* B_n is the remaining amount to repay on the load for term n.
* $A = P(A/P, i, N)$

Add-On Loans

In an add on load, the total interest rate is precalculated and added onto the principal, and this sum is them payed in equal installments. The interest rate quoted is the **add-on interest**. For a loan of P dollars for N years with an add-on rate of i, the following holds:

$$\text{Total add-on interest } = PiN$$

$$\text{Principal plus add-on interest } = P + PiN = P(1 + iN)$$

$$\text{Monthly installments } = \frac{P(1 + iN)}{12N}$$

Mortgages

A mortgage is a special type of loan used to purchase a property. The borrower is the mortgagor, and the lender is the mortgagee. The amount of the load is the *principle*. The difference between the price of the property and the amount one owes on the mortgage is the *purchaser's equity.* There are multiple types of mortgages:

1. Fixed-rate mortgages offer loans whose interest rates are fixed over the period of the contract.

2. Variable-rate mortgages offer interest rates that fluctuate with market conditions.

3. Conventional mortgages are any type of home buyer's loan that is not offered or secured by a government entity. The purchaser is responsible for 20% as a down payment.

4. High-ratio mortgages are when the mortgagor have less than a certain amount of down payment to put towards the purchase of a home. The purchaser can make a down payment as low as 5%. These loans must be insured.

5. Collateral mortgages are re-advanceable mortgage products.

There are 7 terms and conditions to consider before signing a mortgage:

1. **Amortization** refers to the number of years it would take to repay the loan in full. This is typically 25 years.

2. **Term of the mortgage** refers to the number of years the legal document covers, at the end of a term one has the option to renew the mortgage or refinance with another bank.

3. **Interest rate** is typically a nominal rate compounded semi-annually. For a variable rate mortgage this number can change between compounding periods.

4. **Open or closed mortgages**. In an open mortgage, you can repay the loan faster than the agreed upon rate, and so the interest rate increases. In a closed mortgage, you cannot pay the bank back faster without a penalty.

5. **Payment schedule** refers to the frequency of payments, typically monthly.

6. **Prepayment privileges** are special circumstances that let you pay some percentage ahead in a closed mortgage.

7. **Portability** refers to the ability of the loan to be transferred to another property without penalty.

Bonds

Bonds are a special form of loan in which the creditor promises to pay a stated amount of interest at specified intervals for a defined period and then to repay the principle at a specified date, known as the maturity date of the bond. The following is some terminology relating to bonds:

* **Par value (or face value)** the amount the bond is issued in. Usually a multiple of $1000
* **Maturity date** the date at which the par is due back to the issuer.
* **Coupon rate** is the interest paid on the par value of the bond as a regular payment.
* **Discount or premium bonds** are bonds that sell at values other than its par value. A discount bond sells below, while a premium bond sells above. The price on has to pay to purchase a bond is its *market value.*
* **Bearer bonds and registered bonds**. A bearer bond does not have the name of the bond owner on it, and the holder of the bond is presumed to be the owner. While a registered bond shows the owner on it.

The yield of maturity represents the actual interest earned from a bond over the holding period, it is the yield that would be realized on a bond or other fixed income security if the bond was held until the maturity date. *The current yield* of a bond is the annual interest earned as a percentage of the market price.

Week VI

Chapter 5: Analysis of Independant Projects

An **independant** project is one in which the cost and benefits does not depend on whether another is chosen. Projects are considered one at a time and can either be accepted or rejected without considering it's affect on other projects. **Mutually exclusive** projects is where one project is excluded if another is chosen, mutually exclusive alternatives (set of projects) compete with one another to be accepted or rejected. Projects in general are usually given a set amount of years to be active.

The **payback period** is the number of years it takes for an investment to be recouped (net 0 profit) when the interest rate is assumed to be 0. The project with the shorter payback period is the preferred investment, and can be calculated by:

$$\text{Payback period} = \frac{\text{First cost}}{\text{Annual savings}}$$

To find the payback period while considering some interest rate i then you need the **Discounted Payback Period**. To calculate this, for each payment period, add on the interest from the previous period's final balance, and subtract the savings. When the cumulative cash flow becomes positive, the time between the start of the investment and that time is the **Discounted Payback Period**. An example of this is as follows:

Say you make an investment of $\$85,000$ at 15% interest which allows you to save a certain amount (visible in the first column of the table below) each year. What is the Discount Payment Period?

Using the table below:

Period	Cash Flow	Cost of Funds	Cumulative Cash Flow
0	$-\$85,000$	0	$-\$85,000$
1	$\$15,000$	$-\$85,000(0.15) = -\$12,750$	$-\$82,750$
2	$\$25,000$	$-\$82,750(0.15) = -\$12,413$	$-\$70,163$
3	$\$35,000$	$-\$70,163(0.15) = -\$10,524$	$-\$45,687$
4	$\$45,000$	$-\$45,687(0.15) = -\$6,853$	$-\$7,540$
5	$\$45,000$	$-\$7,540(0.15) = -\$1,131$	$\$36,329$

Evidently the cumulative cashflow becomes positive between year 4 and 5, and so the discounted payback period is 5 years.

The **Net Present Worth** (NPW) is the present worth of all cashflows in an investment. An investment firm has a **minimum accepted rate of return** (MARR) that it wants to make on it's investments. The NPW is calculated as a function of the MARR, as in $NPW(MARR)$ usually written as $NPW(i)$. The **Net Future Worth** (NFW) is similarly the future worth of all cashflows in the project, and it also calculated as a function of the MARR. Finally, the **annual equivalent worth** (AE) is the equivalent annual payment or receipt that would be made during the course of the investment, and is also a function of the MARR. The investment firm will make a decision on whether to accept or reject an investment based on the following criteria:

* If $NPW(i) > 0$ then accept the investment.
* If $NPW(i) = 0$ then remain indifferent.
* If $NPW(i) < 0$ then reject the investment.
* If $NFW(i) > 0$ then accept the investment.
* If $NFW(i) = 0$ then remain indifferent.
* If $NFW(i) < 0$ then reject the investment.
* If $AE(i) > 0$ then accept the investment.
* If $AE(i) = 0$ then remain indifferent.
* If $AE(i) < 0$ then reject the investment.

The **MARR** is determined by considering two factors:

1. The **Cost of Capital** which is the required return necessary to make an investment project worthwhile, as in what rate of return would the firm receive if they invested their money somewhere else with similar risk.

2. The **Risk Premium** which is the additional risk associated with the project.

Annual Equivalent Cost (AEC) is also used and is similar to the (AE) method, however only considered cost, which includes the cost of the capital (interest on loan), and the operating costs.

Capital Recovery (CR) is the equivalent annual amount that the project must earn each year to recover the investment plus some stated rate of return. Capital recovery calculations only take into account the initial investment (P) and the salvage value (S), and can be calculated in general by using:

$$CR(i) = P(A/P, i\%, N) - S(A/F, i\%, N) = (P - S)(A/P, i\%, N) + S(i\%)$$

The **Unit Profit/Cost** is the cashflow as a result of each unit produced during an investment, for example if an investment produces 1000 tools, what is the cashflow as a result of each single tool. In general an asset is a group of units, and a unit is something that costs/makes money. There are 5 steps to calculate unit profit/cost:

1. **Step 1**: Determine the number of units to be produced each year over the life of the asset.

2. **Step 2**: Identify the cash flow series associated with production over the life of the asset.

3. **Step 3**: Calculate the NPW of the project cash flow series at a given interest rate and then determine the AEW.

4. **Step 4:** Divide the AEW by the number of units produced each year.

A **Simple Investment** is a project with only one sign change in the net cash flow. As in for some amount of time the net cash flow is negative, and it switches to positive at some later time and stays positive. A **Non-Simple Investment** is the opposite.

Rate of Return (ROR) i^*

There is 3 separate definitions for the same concept of ROR:

* **Definition 1**: The interest rate earned on the unpaid balance of an amortized load.

* **Definition 2**: The break even interest rate i^* that equates the present worth of a project's cash outflows to the present worth of its cash inflows. As in:

$$NPW(i^*) = 0$$

- **Definition 3**: The *Internal ROR* is the interest rate earned on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance will be 0. This is the return that a company would earn if it invested in itself, rather than investing that money elsewhere.

I feel that the second definition is the best one to work with.

In order to find the ROR (i^*) you need to equate the NPW(i^*), NFW(i^*), or the AE(i^*) to 0 and solve for i^*. This becomes almost impossible to find a closed form solution for (called **direct substitution method**) if there are more than two cashflows.

In this case you could either use a **computer method**, or you can use the method of **trial-and-error** in guessing certain values for i^* and adjusting to get a closer and closer approximation.

The *Internal ROR* (IRR) from Definition 3 can be used to determine whether to accept a project by the following rules:

- If $IRR > MARR$ then accept the investment.
- If $IRR = MARR$ then remain indifferent.
- If $IRR < MARR$ then reject the investment.

Note: week VII had no content.

Week VIII

Chapter 6: Comparing Mutually Exclusive Alternatives

When comparing mutually exclusive projects, a project is excluded if another is chosen. Something to note is the alternative of "doing nothing" is always an option, which has a rate of return equal to MARR. A **Service project** is a project whose revenues do not depend on the choice of alternative, as in all alternatives produce the same revenue. A **Revenue Project** is a project whose revenues depend on the choice of alternatives. An additional consideration to make in this type of project is what will be done with the difference in investment funds if the firm goes with a project that requires a smaller investment. The not invested funds will continue to earn interest at the MARR.

In general, a pair of investments called A and B can be related by the equation:

$$B = A + (B - A)$$

As in the investment B which is defined to be the more costly option, is the sum of A and the difference between A and B. $B - A$ can be considered it's own investment option even though of course it cannot be chosen. This is called **Incremental Analysis**. This analysis method is used to justify the additional investment needed for the more costly of the mutually exclusive options. Using the investment $B - A$ we can decide what course to choose between A and B, as in:

* If $IRR_{B-A} > MARR$, select option B.
* If $IRR_{B-A} = MARR$, select either option.
* If $IRR_{B-A} < MARR$, select option A.

If the "do nothing" alternative is allowed, then the selected option's IRR must be greater than the MARR, otherwise do nothing. PW, FW, and AEW analysis still apply for the $B - A$ option as in:

* If $PW_{B-A}(MARR) > 0$, select option B.
* If $PW_{B-A}(MARR) = 0$, select either option.
* If $PW_{B-A}(MARR) < 0$, select option A.
* If $FW_{B-A}(MARR) > 0$, select option B.
* If $FW_{B-A}(MARR) = 0$, select either option.
* If $FW_{B-A}(MARR) < 0$, select option A.
* If $AEW_{B-A}(MARR) > 0$, select option B.
* If $AEW_{B-A}(MARR) = 0$, select either option.
* If $AEW_{B-A}(MARR) < 0$, select option A.

When examining multiple investment opportunities, you can always disregard options using the IRR vs MARR criteria, and then compare investments in pairs.

It is **not** necessary to use incremental analysis, you can compare two mutually exclusive projects using NPW over the same time period. Mutually exclusive events must be compared over the same analysis period, which is the time span over which the economic effects of an investment will be evaluated. In general, the analysis period should be chosen to cover the required service period of the investment, If this is true, then a simple present worth analysis will work. If this is not true, then specific adjustments need to be made:

1. If the analysis period is shorter than the project's life (service period):
   * Evaluate the salvage value at the end of the required service period.
   * Compute the PW for each project over the required service period.
   * Ignore all cash flows after the required service period.
2. If the analysis period is longer than the project's life (service period):
   * Come up with replacement projects that match or exceed the required service period. This will be given in the question as something that can be leased for a certain yearly fee for a set amount of years to fill in the time.

* Compute the PW for each project over the required service period.
* If one of the projects service life coincides with the required service period, then no leasing option is needed.

3. If the analysis period is not specified:

    * Come up with replacement projects that serve out the least common multiple period. Repeat the projects until the LCM of the service periods is met.
    * Compute the PW for each project over the required service period. Then bring them all back to year 0.
    * The following is true for infinite horizons project plans:

$$P = AEW \cdot MARR$$

To decide whether a firm will make a part in-house or to buy it is the **Make-or-Buy Decision**:

1. Determine the time span for which the part will be needed.
2. Determine the annual quantity of the part.
3. Determine the equipment, manpower, and all other resources required to make the part.
4. Estimate the next cash flow associated with the "make" option over the same time span.
5. Compute the annual equivalent cost of producing the part.
6. Computer the unit cost of making the part by dividing the annual equivalent cost by the required annual volume.
7. Choose the option with the minimum unit cost.

Week IX

Chapter 11: Replacement Decisions

This chapter focuses on analyzing the decision to replace a machine or operation with another, the following questions will be asked and answered: Is is beneficial to replace? When is the most efficient time to replace? How long should the replacement be kept for before replacing again?

Basic Concepts and Terminology

* **Defender:** is the existing asset possibly being replaced.
* **Challenger:** is the best available candidate.
* **Current Market Value:** is the value to use in preparing a defender's economic analysis. This is the amount of money that the firm could sell the defender for today. This is considered to be a payment when analyzing the worth of the defender because you essentially have that money in your hand, and if you choose to keep the defender you are *spending/losing* that amount.
* **Sunk Cost:** is all the past costs relating to the defender, which cannot be used make any future decisions. It represents money that is gone, and no present action can recover them.
* **Operating Cost:** is the sum of all the costs of operating an asset, usually annually.
* **Original Cost:** is the original payment made for the defending asset. (Irrelevant for replacement analysis)
* **Trade-in Allowance:** is the amount of money the asset vendor is allowing you to get by trading in the defender for the challenger, even if the defender has not been used to it's full lifespan. The value is sometimes the entire *current market value.*
* **Repair Cost:** is the amount spent on repairs for the defender in the past. (Irrelevant for replacement analysis)
* **Future Salvage Value:** is the future salvage value of the defender at the end of it's lifespan.

The simplest way to consider replacing an asset is the *Opportunity Cost Approach*, in which essentially you just find the **NPW** of both the defender over it's lifespan, and the challenger over it's lifespan and compare the costs. Keeping in mind that the current market value of the defender is a cash outflow at year 0.

Economic Service Life

The economic service life is the remaining useful life of an asset that results in the minimum annual equivalent cost. The cost of owning and operating an asset is split into two categories called the **Capital Cost** and the **Operating Costs**.

The capital costs considers the initial investment (P) in competing assets *today* as well as the salvage value. S_N is used to denote the salvage value at the end of ownership period of N years. Capital recovery considers the principal cost and the salvage value only as annual equivalent payments. The principal for the challenger is just the purchase price, and for the defender it is the opportunity cost (current market value). The annual equivalent of capital costs, called the capital recovery cost ($CR(i)$) can be calculated using the following:

$$CR(i) = P(A/P, i, N) - S_N(A/F, i, N)$$

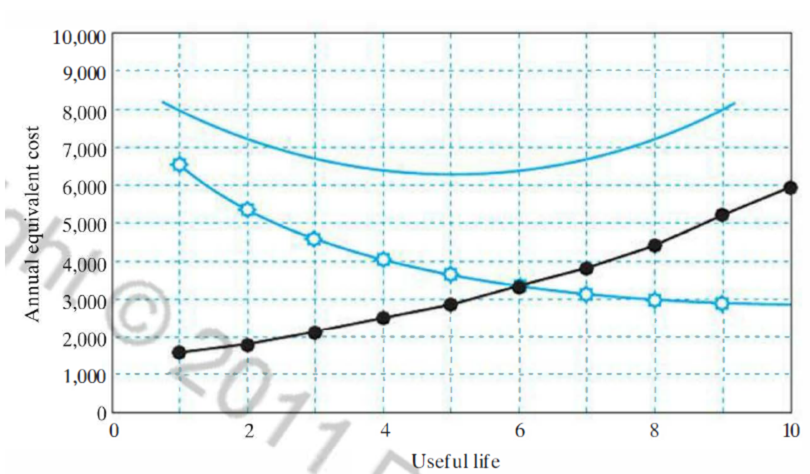
The annual equivalent of the operating costs ($OC(i)$) over a lifespan of N years, where OC_n is the operating costs in year n is:

$$OC(i) = \left(\sum_{n=1}^{N} OC_n(P/F, i, n) \right) (A/P, i, N)$$

... and finally the total annual equivalent costs of owning and operating an asset ($AEC(i)$) is:

$$AEC(i) = CR(i) + OC(i)$$

In general, operating costs increase as the years go on due to the higher chances of failure and need for repair/maintenance. For the capital recovery function, as long as the salvage value is less than the initial cost, the capital recovery cost is a decreasing function. The longer we keep an asset, the lower the capital recovery cost becomes. These ideas lend themselves to the following graph:



- $AEC(i)$ is the top curve.
- $CR(i)$ is the decreasing blue curve.
- $OC(i)$ is the black curve.

There is a unique minimum point on the graph of $AEC(i)$ which occurs at the year $n^* = 5$ in the example used for this graph. n^* is the year the minimizes the $AEC(i)$. To find n^*, evaluate $AEC(i)$ for each year the expected service life of the project, and the year with the smallest value is the n^*. The economic service life is then used for analysis.

Decision Frameworks

The **planning horizon** is the service period required from the defender, and then a sequence of future challengers. This would either be **infinite** (in which you would use the annual equivalent cost method) or it could be **finite** (in which you would use the present equivalent cost method). No technological improvement is anticipated over the planning horizon.

If an asset of identical price and features can be purchased repeatedly over an indefinite period, we would always replace the asset at the end of it's economic service life.

The notation used for planning horizons is the following:

$$(j_0, n_0), (j_1, n_1), (j_2, n_2), \ldots, (j_k, n_k)$$

... where each j_k represents some challenger used for n_k years, except for j_0 which is the defender.

The following is a general strategy under an infinite planning horizon:

1. Compute the economic lives of both the defender (N_{D^*}) and the challenger (N_{C^*}), and compute AEC_{D^*} and AEC_{C^*} as the annual equivalent costs for the defender and challenger at their respective economic lives. This means minimize the $AEC(i)$ function for both the challenger and then use that AEC and n value to do the replacement analysis.

2. Compare AEC_{D^*} and AEC_{C^*}, if $AEC_{D^*} > AEC_{C^*}$ then the challenger should be chosen now, otherwise the challenger should not be chosen now, and the defender should continue to be used at least for the duration of its economic life.

3. When should the defender be replaced? The defender needs to be kept until its economic life is over, then:

   ∗ Compute the cost of running the defender for one more year. If this cost is greater than AEC_{C^*}, the defender should be replaced at the end of it's economic life.

   ∗ Compute the cost of running the defender for the second year after its economic service life. If this cost is greater than AEC_{C^*}, the defender should be replaced one year after the end of it's economic life.

   ∗ Compute the cost of running the defender for n^{th} year after its economic service life. If this cost is greater than AEC_{C^*}, the defender should be replaced $n-1$ years after the end of it's economic life.

The process described in part 3 above is called **Marginal Analysis**. This procedure assumes there is no new challenger, if there *is* then repeat the whole process starting at the time of the new best challenger. In the marginal analysis when you are evaluating the AEC for an additional year, you only consider 3 things:

* The salvage value which is now the opportunity cost (as a cost).
* The salvage value at the end of the next year (as a receipt).
* The payment made at the end of the next year (as a cost).

Annual cash flow analysis with unequal service life alternatives are not valid under infinite planning horizons.

The procedure for solving finite planning horizon is to establish all reasonable replacement patterns and then use the PW or PEC value for the planning period to select the most economical pattern.

Week X

Chapter 8: Asset Depreciation

Depreciation is the gradual decrease in utility of fixed assets with use and time. Mathematically that is:

$$\text{Economic Depreciation} = \text{Purchase Price} - \text{Market Value}$$

Physical Depreciation is defined as a reduction in an asset's capacity to perform its intended service due to physical impairment. **Function Depreciation** occurs as a result of changes in the organization or in technology that decrease or eliminate the need for an asset. **Accounting Depreciation** is the cost of the acquisition of the machine spread over the years which the machine is in service. This is the usual functional use of depreciation in real life, and so it is sometimes called simply **asset depreciation**. This is our main concern for this chapter.

Accounting depreciation is based on the **matching concept**, which is how it generally reflects the actual economic depreciation of the asset. A fraction of the cost of the asset is chargeable as an expense in each of the accounting periods in which the asset provides a service, this charge is meant to represent the percentage of the value utilized in the given period. A depreciable asset is a property for which a firm may take depreciation deductions against income. A depreciable asset must:

* Be used in business or held for production of income.
* Have a definite useful life.
* Must wear out.

There are four things needed to calculate depreciation:

1. The cost basis of the asset, as in what is the general cost of the asset?
    * This is the total cost that is claimed as an expense over the asset's life. Including the initial cost and all other incidental expenses.

Sales tax however, does not count towards to cost bases of the asset. The **book value** of an asset is the cost basis minus the total accumulated depreciation at some time in the future after purchase. A **trade-in** purchase must consider the difference between the cost basis and the total accumulated depreciation (book value). If the trade-in allowance exceeds the book value, the difference (this difference is called **unrecognized gains**) needs to be subtracted from the cost basis of the new asset. If the opposite is true (**unrecognized loss**) the difference should be added to the cost basis for the new asset.

2. The depreciable life of the asset (in years).

   * **Useful life** is an estimate of the duration over which the asset is expected to fulfill its intended service life. This useful life of the asset is the depreciable life of the asset.

3. The salvage value of the asset, as in the estimated market value at the end of its depreciable life.

   * The asset's value at the end of its life. The amount recovered through sale, or net salvage of all costs incurred for disposal. Assumed to be zero for very long lived assets. The estimated salvage values will not equal the projected book value at the end of the depreciable life.

4. The method of it's depreciation.

   * A firm calculates depreciation in two different ways: If it is intended for financial reports such as a balance sheet or income tax statement they use the **Book Depreciation Method**. If it is for the Canada Revenue Agency for the purpose of calculating taxes they use the **Capital Cost Allowance**. Generally book depreciation is used to report to stockholders and outside parties, while capital cost allowance is used for income tax statements.

Book Depreciation

There are three methods to calculate the periodic depreciation allowances.

The **straight line method** is used for an asset that depreciates uniformly over its expected service life. To calculate depreciation:

$$D_n = \frac{(P - S)}{N}$$

Where:

* D_n is the depreciation charge during year n.
* P is the cost of the asset including installation expenses.

* S is the salvage value at the end of the asset's useful life.
* N is the useful life of the asset in years.

This is used to calculate the depreciation of an asset in some year n. The following formula describes the **book value** of an asset at the end of n years:

$$B_n = P - (D_1 + D_2 + \cdots + D_n) = P - \frac{n(P-S)}{N}$$

The **Accelerated Methods** are used when a stream of services provided by a fixed asset may decrease over time. This is because for example, mechanical efficiency is greater when the asset is new compared to when it is at the end of it's expected service life. These methods charge a larger fraction of the cost as an expense of the early years than of the later years. The two most widely used accelerated methods are what follow.

The **Declining Balance Method (DB)** allocates a fixed fraction (d) of the beginning book balance each year, defined by:

$$d = \frac{multiplier}{N}$$

When this multiplier is 1 then the method is called **single declining balance method**. This method is the basis for most CCA rates. The multiplier could also be 1.5, 2... and so on. As N increases, d decreases, which gets us the result we want.

This factor d can thus be used to determine depreciation charges for a given year:

$$D_n = dP(1-d)^{n-1} \text{ for } n \geq 1$$

You can also compute the total declining balance depreciation (TDB) at the end of n years:

$$TDB_n = D_1 + D_2 + \cdots + D_n = P[1 - (1-d)^n]$$

... and the book value (B_n) at the end of n years:

$$B_n = P - TDB_n = P(1-d)^n$$

The salvage value (S) must be estimated independently of depreciation analysis, and rarely is equal to the book value at the end of the asset's expected service life. There are two cases that can occur:

1. **Case 1:** $B_n > S$

    * Here, we have not depreciated the entire cost of the asset. If a firm wants to reduce its book value to its salvage value as quickly as possible, it would switch from DB to SL whenever SL would result in larger depreciation charges. In general, if depreciation by DB in any year is less than (or equal to) what it would be by SL, then we

should switch to and remain with the SL method for the remaining duration of the project's depreciable life. Mathematically, switch to SL in year n where the following holds:

$$dB_{n-1} \leq \frac{B_{n-1} - S}{N - n + 1}$$

2. **Case 2:** $B_n < S$

   * Here, the asset has been depreciated past its salvage value. In this situation you just stop depreciating when:

$$B_n = S$$

The **Sum-of-Years'-Digits (SOYD) Method** is another accelerated method for allocating the cost of an asset's depreciation. SOYD results in larger depreciation charges during the early years of an asset's life, and smaller charges as the asset reaches the end of its estimated useful life. SOYD guarantees $B_n = S$ at the end of the asset's expected useful service life. This method uses:

$$SOYD = 1 + 2 + \cdots + N = \frac{N(N+1)}{2}$$

In general to find the depreciation in year n we use:

$$D_n = \frac{N - n + 1}{SOYD}(P - S)$$

Units-of-Production (UP) Method

The depreciation method views the asset as a consisting of a bundle of service units, not assumed to be consumed in a time-phase pattern. The cost of each service of the asset is the net cost of the asset divided by the total number of such units, and used to find the depreciation charge for a period. The depreciation in any year is given by:

$$D_n = \frac{\text{Service units consumed during year } n}{\text{Total serviced units}}(P - S)$$

This method gives a more accurate picture of machine usage since depreciation charges are made proportional to the ratio of actual output to the total expected output.

Capital Cost Allowance for Income Tax

Capital Cost Allowance refers to a portion of the acquisition cost of an asset that may be deducted from income each year, which may lower payable taxes. There is a one to one correspondence between terms used in book depreciation to words used in tax depreciation:

Book Depreciation Term	Tax Depreciation Term
Asset	Property
Depreciation	Capital Cost Allowance
Cost Basis	Capital Cost
Book Value	Undepreciated Capital Cost
Salvage Value	Proceeds of Disposition

... why don't we just use the same words if they mean the same thing? Good question.

Tax depreciation has two important differences from book depreciation. First of all there is only one method to tax depreciation to ensure fairness, and secondly most property items are depreciated in groups called *classes* or *asset pools*. The property in most classes must be depreciated using the declining balance method, or sometimes a specified SL method. To limit maximum allowance, the tax rules specify a percentage rate called the *CCA Rate* (d) for each of the classes. A company may claim up to the maximum allowance in each class in each year.

The earliest taxation year in which CCA may be claimed for property is when it becomes **available-for-use**. For buildings this means when the building uses all or substantially all of the building for its intended purpose. And for non buildings that means when the property is first used to produce income. No CCA can be claimed for property in the year it is acquired, or in the following year, unless it is available-for-use. To calculate CCA, a firm must fill out a *Schedule 8 form* where each depreciable asset and its DB depreciation is a line of the form. The total CCA is the sum of all the individual CCAs. The CCA Rate for each class can be found in a CRA table, these values change year-by-year.

The **50% Rule** also knows as the half-year convention limits the CCA of new property acquired during the year to 50% of what it would otherwise be. The rule applies to **net acquisitions** in each class, equal to the amount by which purchases dispositions during the year.

Without the 50% rule:

$$CCA_1 = Pd$$

$$CCA_n = Pd(1-d)^{n-1}$$

$$U_n = P(1-d)^n$$

... recall that in tax depreciation **U**ndepreciated capital cost means book value. Also recall that $d = 1/N$ for single declining balance method.

With the 50% rule:

$$CCA_1 = P \cdot \frac{d}{2}$$

$$CCA_n = Pd(1-d/2)(1-d)^{n-2}$$

$$U_n = P(1-d/2)(1-d)^{n-1}$$

CCA for Individual Projects

There are three assumptions made for calculating CCA of new assets on a project basis:

1. Ignore any activity in the class from outside the project, which means you can apply the 50% rule to a new asset in the project, and allowing the asset to fully depreciate every year of its useful life.
2. The **disposal tax effect** applies and is based solely on the difference $U-S$ in the year of disposal, as in the discrepancy between the book value at the end of the expected useful life of the asset and the actual salvage value. A gain relative to UCC (if $S > U$) gets taxed, while a relative loss from sale (if $S < U$) results in tax savings.
3. Property is disposed of at the end of the year.

Revisions of Depreciation Due to Additions or Alterations

If any major alterations, repairs, or additions are made to the asset that may extend the life of the asset or may increase the originally estimated salvage value of the asset are made then a revised estimate of useful life should be made, and the periodic depreciation expense should be updated accordingly.

For **Book Depreciation** you simply revise the current book value and allocate this cost over the remaining years of useful life.

For **Tax Depreciation** the property that has been altered is treated as new appreciable property acquisitions. The CCA class that applies to the altered property is the class that would apply to the original property if it were made available-for-use at the same time as the alteration. A part added to a depreciable property which requires regular replacement is not considered a capital expenditure.

Week XI

Chapter 14: Inflation and Its Impact on Project Cash Flows

Inflation is defined as the loss in the purchasing power of money over time, which means that the same dollar amount buys less of an item over time. **Deflation** is the opposite, which is far less common. A measure called the **Consumer Price Index** (CPI) is used to measure a relative decline in purchasing power based on a typical *market basket* of goods and services. Mathematically it is related by:

$$CPI_{\text{any year}} = \frac{Cost_{\text{any year}}}{Cost_{\text{base year}}} \times 100$$

This base period is used to calculate the inflation relative to that base year. Consumer product inflation rate can be calculated between two years as:

$$\frac{CPI_n - CPI_{base}}{CPI_{base}}$$

The Industrial Product Price Index (IPPI) is a type of inflationary indicator to evaluate wholesale price levels in the economy.

The **average inflation rate** is a way to account for the effect of varying yearly inflation rates all compounding on top of each other over a period of several years. Just find an interest rate that gets you the final result from the principal. The **General Inflation Rate** ($\bar{f}$) which is an average inflation rate calculated on the basis of CPI for all items in the market basket. Mathematically that is:

$$\bar{f} = \left[\frac{CPI_n}{CPI_0}\right]^{1/n} - 1$$

Where:

* $\bar{f}$ is the general inflation rate.
* CPI_n is the consumer price index at the end of period n.
* CPI_0 is the consumer price index for the base period.

If we want to know the general inflation rate for some period of consecutive years we use the following formula:

$$\bar{f}_n = \frac{CPI_n - CPI_{n-1}}{CPI_{n-1}}$$

Specific inflation rate (f_j) is the rate based on a specific segments of the economy.

Actual versus Constant Dollars

Actual (current) dollars (A_n) are estimates of future cash flows for year n that take into account any anticipated changes in amounts caused by inflation. These are dollars that react to inflation.

Constant (real) dollars (A'_n) represent constant purchasing power that is independent of the passage of time and inflation. Where inflationary effects are assumed when estimating cash flows, those cash flows can be converted to constant dollars by adjustment with some general inflation rate. Base year is time 0. Essentially these dollars represent dollar amounts expressed in terms of purchasing power of the base year.

To convert from constant dollars to actual dollars:

$$A_n = A'_n(1+\bar{f})^n = A'_n(F/P, \bar{f}, n)$$

To convert from actual dollars to constant dollars:

$$A'_n = A_n(1+\bar{f})^{-n} = A_n(P/F, \bar{f}, n)$$

Equivalence Calculations Under Inflation

Due to the changes in purchasing power as well as changes in earning power (inflation), we use either *constant-dollar analysis* or *actual-dollar analysis* to do equivalence calculations. There are two types of inflation rates used in estimating cash flows: **Market Interest Rate** (i) which takes into account the combined effects of the earning value of capital, and any anticipated inflation. This rate is also called the *inflation-adjusted MARR.* The **Inflation-free interest rate** (i') is an estimate of the true earning power of money when the inflation effects have been removed, this is also called the *real interest rate.* In the absence of inflation, the market interest rate is the same as the inflation-free interest rate. Note that equivalence calculations will be done on sets of cash flows that are either all in actual dollars or all in constant dollars, if it is mixed then convert to one.

Constant-Dollar Analysis is the analysis method used when all cash flows are in constant dollars. To compute the present worth of constant dollars occurring in year n after the base year (year 0) we use:

$$P_n = \frac{A'_n}{(1+i')^n}$$

In general you can find the present worth of a project which is described in constant dollars by simply finding NPW with i' as the interest rate.

Actual-Dollar Analysis is the analysis method used when all cash flows are in actual dollars. There are two sub-methods called the **deflation method** and the **adjusted-discount method**.

The **Deflation Method** requires two steps to convert actual dollars into equivalent present-worth dollars. *First* you convert all actual dollars into equivalent constant dollars by discounting by the general inflation rate, which removes the inflationary effect. Then you use i' to find the equivalent present worth.

The **Adjusted-Discount Method** performs deflation and discounting in one step. The most important relationship as a result of this method is:

$$i = i' + \bar{f} + i'\bar{f}$$

... which tells us that the market interest rate is a function of i' and $\bar{f}$. To determine the present worth, find NPW using i defined above as your interest rate, as in:

$$P_n = \frac{A_n}{(1+i)^n}$$

Effects of Inflation on Project Cash Flows

Capital cost allowances and interest expenses are always given in actual dollars. CCAs are calculated on some base-year purchase amount, and do not increase over time to keep pace with inflation. Salvage values can increase with the general inflation rate. The goal of this section is to be able to fill out the following table using the knowledge we've built up from the course:

	N=0	N=1	N=2	N=3	N=4	N=5	N=6
INCOME STATEMENT							
Revenues							
Expenses							
O&M							
Debt Interest							
CCA (%)							
Taxable Net Income							
Income Taxes (%)							
Net Income							
CASH FLOW STATEMENT							
Operating Activity							
Net Income							
CCA							
Investment Activity							
Investment							
Working Capital							
Salvage							
Disposal Tax Effect							
Financing Activity							
Borrowed Funds							
Principal Repayment							
Net Cash Flow							

The following equations are useful for completing the income tables, in order of how they appear in the rows above (*Note:* any row beginning in ($\bar{f}$) inflates):

($\bar{f}$) Revenues: Given in the question.

Expenses: Nothing goes in this row directly.

$(\bar{f})$ O&M: Given in the question.

Debit Interest: $I_n = B_{n-1} \cdot i(\text{of loan})$

CCA: Use CCA formulas.

Taxable Income = Revenues−Total Expenses−CCA(CCA Rate)−Debt Interest

Income Taxes = (Taxable Income) × (Income Tax Rate)

Net Income = Taxable Income − Income Taxes

Investment: Given in the question.

Working Capital:

$(\bar{f})$ Salvage: Given in the question.

The **Disposal Tax Effect** deals with the difference between final book value and salvage value. In general:

$$DTE = (U_N - S) \times (\text{Tax Rate})$$

This amount needs to be subtracted from the net cash flow of the final year.

Borrowed Funds: Amount of the principal that was borrowed in a loan.

Principal Repayment = Loan Annuity − Debt Interest

Net Cash Flow = Sum of all rows down after the header "CASH FLOW STATEMENT"

If certain parts of a project inflate at different rates, then they all need to be individually accounted for. **Inflation Tax** is defined by:

$$\text{Inflation Tax} = NPW_{\text{without inflation}} - NPW_{\text{with inflation}}$$

The repayment of a loan is based on the contract amount, the payment size does not change with inflation. Interest expenses are usually already in actual dollars.

From the net cash flow row at the bottom, you can do present worth analysis using the appropriate interest rate (i), or rate of return analysis.